

Wire Suspension Bridge Analyzed Considering Suspenders Deformation

Wenjie Niu

Hydraulic structure and environment engineering research institute, Zhejiang University

Hangzhou City, China

nwj1982@sohu.com

Abstract—Elastic theory is employed for short span suspension bridges while deflection theory and finite element theory are used for long span suspension bridges. But the tension deformations of the suspenders are usually ignored in the elastic theory when to compute the internal force of small span suspension bridge. In this paper, a flexibility method considering suspenders deformation was proposed to calculate the internal load of a wire suspension bridge. Its span is less than 200 meters and can be analyzed with elastic theory. Results indicate that the horizontal cable tension force of wire suspension bridge by flexibility method ignoring the suspenders deformations is 0.01671% bigger than the real redundant bar force by flexibility method considering the suspenders deformations. A new equation considering suspenders deformation based on the old common equation was established to find the more accurate horizontal tension force at cable midpoint.

Keywords—Flexibility Method; Wire Suspension Bridge; Tension Deformations of the Suspenders; Elastic Theory

I. INTRODUCTION

Suspension bridges built between 19th century end and 20th century start such as Brooklyn Bridge are analyzed with elastic theory. In the case of very rigid stiffening trusses, the deflections of cable and simply supported beam η can be neglected [1]. Influence line equation for horizontal tension force at wire suspension bridge cable midpoint in Fig. 1 which is widely used in computing and designing short span wire suspension bridges has been determined as [2]

$$H_L = \left(\frac{1}{EI} \int_0^l M_0 y dx \right) / \left(\frac{L_E}{A_c E_c} + \frac{l_1 + l_2}{A_c E_c \cos^3 \phi_0} + \frac{1}{EI} \int_0^l y^2 dx \right) \quad (1)$$

Where d is the defined distance unit (the distance between two adjacent suspenders), E_c is the Elastic module of cable and suspenders, A_c is the Area of cables and suspenders, $E_c A_c$ is the Extensional rigidity, E is the Elastic module of main beam, I is the Moment of inertia and EI is the Bending stiffness. ϕ_0 is the Angle formed by the tangent at the cable end and the horizontal line and equals the Angle formed by Anchorage cable straight line and the horizontal line.

In the common computing method to wire suspension bridge, the tension deformations of the suspenders are usually ignored in the elastic theory when to compute the internal force of suspension bridge [2]. For example, it can be seen that the tension deformations of suspenders are ignored in (1). It is necessary to give the new method considering suspenders deformation and investigate how much accuracy equation (1) can get for determining the horizontal cable tension force.

In this work, a program developed from flexibility method considering suspenders deformation was introduced here and used for one span and two hinge joint wire suspension bridge [3, 4]. Elastic theory was used to determine the internal force of suspension bridge.

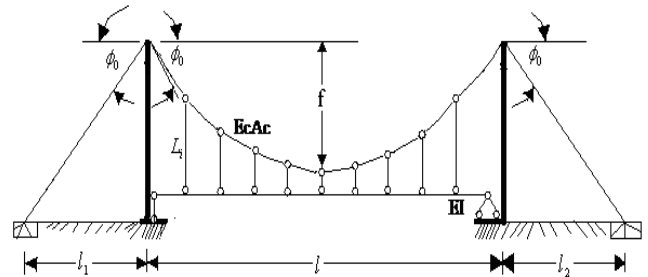


Figure 1. Layout of Wire Suspension Bridge

II. WIRE SUSPENSION BRIDGE CONSIDERING SUSPENDERS DEFORMATION

A. Influence Line Considering Suspenders Deformation

Considering suspenders deformation, influence line for horizontal cable tension force of wire suspension bridge in Fig. 1 can be derived as

$$H_{Lcsd} = \frac{\frac{1}{EI} \int_0^l M_0 y dx}{\frac{L_E}{A_c E_c} + \frac{l_1 + l_2}{A_c E_c \cos^3 \phi_0} + \frac{1}{EI} \int_0^l y^2 dx + \sum_{i=1}^n \frac{(qd)^2}{A_c E_c} L_i} \quad (2)$$

$$q = \frac{8f}{l^2} \quad (3)$$

In (2) and (3), q denotes intensity of the vertical load on the cable by suspenders induced by horizontal unit redundant force at the midpoint of main cable when analyzed with flexibility method. Value L_i represents length of every suspender.

B. Relationship between Influence Line not Considering Suspenders Deformation and Considering Suspenders Deformation

Combining (1) and (2), the relationship between influence line not considering suspenders deformation and considering suspenders deformation can be written as the ratio below

$$\frac{H_L}{H_{Lcsd}} = 1 + \frac{\sum_{i=1}^n \frac{(qd)^2}{A_c E_c} L_i}{\frac{L_E}{A_c E_c} + \frac{l_1 + l_2}{A_c E_c \cos^3 \phi_0} + \frac{1}{EI} \int_0^l y^2 dx} \quad (4)$$

III. A WIRE SUSPENSION BRIDGE EXAMPLE

Wire suspension bridges are designed and analyzed by many researchers [5, 13]. The wire suspension bridge (Fig. 2) parameters were given as (Institute of Bridge Science of Bridge Engineering Department of Railways Ministry 1996):

$$d=7\text{m}; l=140\text{m}; l_1=60.30\text{m}; l_2=53.90\text{m}; \phi = \frac{f}{l}=1/9; f=15.5\text{m}; E_c=1.5 \times 10^{11} \text{N/m}^2; A_c=0.0221\text{m}^2; E_c A_c=3.32 \times 10^9 \text{N}; E=2.1 \times 10^{11} \text{N/m}^2; I=0.1330 \text{m}^2; EI=2.790 \times 10^{10} \text{N}\cdot\text{m}^2.$$

The main cable mathematical equation is

$$y = 4f \frac{x}{l} \left(1 - \frac{x}{l}\right). \quad (5)$$

Insert the parameters above into (5), one can find that equation below can represent the main cable curve in Fig. 2.

$$y = 62.20 \frac{x}{l} \left(1 - \frac{x}{l}\right). \quad (6)$$

From Fig. 2, one can observe that

$$\tan \phi_0 = \left. \frac{dy}{dx} \right|_{x=0}. \quad (7)$$

Combine (5) and (7), the following results can be obtained.

$$\tan \phi_0 = 4 \left(\frac{f}{l} \right) = \frac{4}{9} = 0.4444. \quad (8)$$

$$\cos \phi_0 = 0.9137. \quad (9)$$

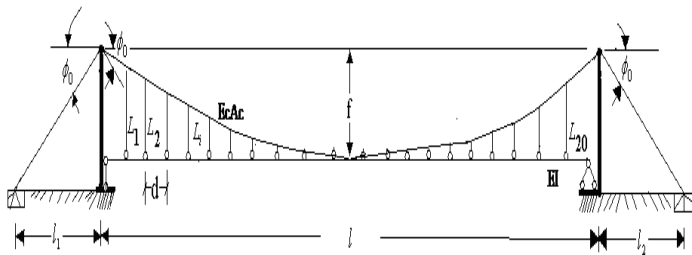


Fig.2 Wire Suspension Bridge to Be Analyzed

IV. RESULTS

Equation (4) can be transformed as

$$\frac{H_L - H_{Lcsd}}{H_{Lcsd}} = \frac{\sum_{i=1}^n \frac{(qd)^2}{A_c E_c} L_i}{\frac{L_E}{A_c E_c} + \frac{l_1 + l_2}{A_c E_c \cos^3 \phi_0} + \frac{1}{EI} \int_0^l y^2 dx}. \quad (10)$$

It means that the influence line derived by (1) deviates from the more accurate value derived by (2). If the above

parameters of wire suspension bridge in section β are inserted into (10), one can get the following results.

$$\frac{H_L - H_{Lcsd}}{H_{Lcsd}} = 0.01671\%. \quad (11)$$

The horizontal tension force at wire suspension bridge cable midpoint in Fig. 2 by flexibility method ignoring suspenders deformation is 0.01671% bigger than the one which is more accurate determined by flexibility method considering suspenders deformation. And this small deviation is always neglected in the practical design.

V. CONCLUSION

Influence line for horizontal tension force at wire suspension bridge cable midpoint was analyzed in this paper. A new equation based on the old common equation was established. The horizontal tension force at wire suspension bridge cable midpoint in Fig. 2 determined by old common equation is 0.01671% bigger than the one which is more accurate determined by flexibility method considering suspenders deformation.

The advantage of the proposed method mainly including (2) and (10) is that (2) can more accurately determine the horizontal tension force at wire suspension bridge cable midpoint while (10) can be called to evaluate how much accuracy the old equation (1) can get for small span wire suspension bridges.

ACKNOWLEDGEMENT

The author is supported with Scientific Research Starting Funds for Introduced Talent in the college of Civil Engineering and Architecture of Zhejiang University in the First Half-year in 2010. The Program Number is 188020-544802[18] and the Authorization Number is 21215. The author is also supported with the 47th batch Chinese postdoctoral scientific fund with a funding serial number as 113000-X91001. The financial help is greatly appreciated.

REFERENCES

- [1] S. P. Timoshenko and D. H. Young, "Theory of Structures, " Second ed., McGraw-Hill Book Company, Singapore: 1965.
- [2] Bridge Science Research Institute, Major Bridge Engineering Bureau, the Ministry of Railways, Suspension Bridge. Science and Technology Document Press, Beijing: 1996.
- [3] J. O. Zhou, Q. B. Zhu, and Z. M. Xu, Structure mechanics. Tongji Univ. Press, Shanghai: 2002.
- [4] K. M. Leet, C. M. Uang, and A. Gilbert, Fundamentals of Structural Analysis. 3rd ed. McGraw-Hill Higher Education, Boston: 2008.
- [5] Z. G. SUN, J. SHI, and Y. N. LI, "Design and Analysis for Health-Diagnosis-Oriented Test Model of Suspension Bridges," Engineering Mechanics, vol. 25(1), pp.192-195, 2008.
- [6] J. G. WANG, H. P. PANG, and X. F. LI, "Catenary Cable Element Method for Cable Curve of Suspension Bridges," Chinese Journal of Applied Mechanics, vol. 25(4), pp.627-631, 2008.
- [7] H. P. PANG, J. G. WANG, and X. F. LI, "Static Analysis of Suspension Bridges Based on the Deflection Theory," Journal of Hefei University of Technology, vol. 33(2), pp.261-265, 2010.
- [8] Y. G. TAN, L. SHI, and Z. ZHANG, "An Analytical Element Method on the Calculation of Suspension Bridges," Journal of Wuhan University of Technology, vol. 33(5), pp.884-887, 2009.
- [9] Y. R. PAN, G. H. DU, and L. C. FAN, "A fine calculation of the geometry and internal force of suspension bridge under dead load," China Journal of Highway and Transport, vol. 13(4), pp.33-36, 2000.

- [10] S. Z. XU, S. H. HE, and Y. T. GAI, "Construction simulation for suspension bridge with non-pre-bias of cable saddle," *Journal of Chang'an University*, vol. 27(1), pp.35-39, 2007.
- [11] H. WANG, A. Q. LI, and C. Q. MIAO, "Finite element model updating and validating of Runyang Suspension Bridge based on SHMS," *Journal of Southeast (English Edition)*, vol. 21(4), pp.474-479, 2005.
- [12] L. SHI, C. C. LIU, Z. ZHANG, and P. J. DU, "Nonlinear stochastic static analysis of long span suspension bridge," *Journal of Dalian University of Technology*, vol.44(3), pp.421-424, 2004.
- [13] J. J. WANG, W. H. YANG, and G. D. LIU, "The Geometrical Nonlinear Analysis of Large -Span Suspension Bridge," *Journal of Hunan University*, vol.25(3), pp.70-73, 1998.